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VARIATION IN THE LEADING FEATURES OF THE
TIDE IN DIFFERENT REGIONS

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VARIATIONS IN THE LEADING FEATURES OF THE TIDES IN DIFFERENT REGIONS.

BY W. BELL DAWSON.

THE tides of the eastern coasts of Canada are very varied in character and thus exemplify several different types. They vary in range from the largest tides of the world, in the Bay of Fundy, to a tide which is almost inappreciable in the middle of the Gulf of St. Lawrence. In this they contrast with the tides on the Atlantic coast of the United States, which have a remarkably uniform character from Cape Cod to Florida. On our Pacific coast, the tide shows other features which supplement those of the Atlantic tides.

The object of this paper is to draw attention to the manner in which the tide is found to have some leading feature in each region, which is evidently dominated by some one element in the moon's movements; such as its declination or its distance or the period of its phases. As all our east coast tides, to their furthest limit in the River St. Lawrence, are propagated from the tide of the open Atlantic, it is evident that the influence of some one lunar factor must become more and more pronounced during the progress of the tide, until it stands out as the dominant feature. In this way, the declination effect may develop in one region, or the distance effect in another, until other features in the tide become relatively unimportant.

This has become clear in carrying out the general method adopted for the Survey of Tides and Currents. A limited number of principal tidal stations were established at strategic points, from which the tides at a large number of harbours and ports of secondary importance are deduced by means of "tidal differences." These are differences in time, and ratios for height, by which the tides at other localities can be computed from the tide tables prepared for the principal stations, which are based upon harmonic analysis.

If the attempt is made to deduce the tide at some harbour from a principal station or port of reference at which the tide is of a different type, large errors may result. For instance, tables are published in which the tides of the whole world are referred to Brest in France, by constant differences of time; but the result for many places is quite unreliable. In the United States tide tables, some of our St. Lawrence and Gulf ports used to be referred to distant tidal stations, which resulted in errors of 1½ hours to 2 hours, early or late, in the time of the tide. As an example, the error in the tide at Pictou, within the Gulf, if calculated from Halifax by the use of a constant difference of time, is shown in the following table.

Date	Time of High Water		Actual Difference	Error with Constant Difference	Remarks
	Pictou h. m.	Halifax h. m.			
1896, July 8	7 10	6 15	0 55	- 1 24	Moon's declination maximum north
" "	8 21 11	18 02	3 09	+ 0 50	
" "	9 8 02	6 50	1 12	- 1 07	
" "	9 22 07	18 55	3 12	+ 0 53	
" "	10 9 00	7 50	1 10	- 1 09	New moon
" "	10 23 15	19 30	3 45	+ 1 26	
" "	11 9 45	8 35	1 10	- 1 09	
" "	11 23 57	20 22	3 35	+ 1 16	
" "	12 10 35	9 15	1 20	- 0 59	

It must not be assumed that it is the distance of a port of reference which necessarily interferes with accuracy in deducing the tide from it by means of a constant difference. It may prove quite possible to apply constants to a port of reference in

another part of the world, provided that the type of the tide is really the same. An attempt on these lines was made for the tides along the St. Lawrence river, in the United States tide tables of ten years ago, before data were secured by this Survey. The tides above Quebec as far as Lake St. Peter were referred to various ports in Burma, Germany, Newfoundland, and Florida; but the results thus obtained were quite unsatisfactory. These ports appeared to be chosen as having the same range as the successive points along the St. Lawrence, which were referred to them; but the mere fact that the range of the tide is the same at two distant places affords no guarantee that the tide is of the same type.

If a region of limited extent is taken, it is usually possible to refer all its tides to a principal station in the region by means of time-differences and ratios for height which are constant, and thus to secure good results. But if it should prove that the type of the tide is undergoing change in its progress through the region, the time-differences with the principal station will no longer be constant. It is usually found that the variation in the difference takes place in the period of one or possibly two of the various lunar months known to astronomers. More usually, the variation which corresponds with some one element in the moon's movement is so strongly marked that all other variations can be ignored in practical calculations.

On the Pacific coast, the solar influence is so large, relatively to the lunar, that the chief variation in the tidal difference is found to be an annual one. When this is allowed for, any variations in the course of the month are relatively insignificant.

From this variation in tidal differences, it appears that when some leading feature begins to manifest itself in the tide, this may continue to develop with its progress through the region in question, till it becomes the dominant characteristic. We will not venture to offer an explanation for these variations and changes in the features of the tide, but will only give examples of them, as they are so remarkably illustrated by our Canadian

tides. We will thus hope to make the facts clear; and will leave the explanation to astronomers with competence to deal with it.

The region may be classified with reference to the lunar element which is found to dominate each, in the period of one of the various lunar months. The lengths of the months which have the most importance from a tidal point of view, are given below in tide-intervals or half lunar days, which is the true tidal unit.

Synodic month of the moon's phases	=	57.06	tide intervals.
Anomalistic month; moon's distance	=	53.24	" "
Tropical or declination-month	=	52.79	" "

These values are necessary as a basis in making an analysis of tidal differences in the period of any of these lunar months.

Anomalistic month.—In all parts of the world, the tides are found to accord with the varying movements and distances of the moon and the sun. In the North Atlantic, where they were first studied, it happens that they are chiefly influenced by the moon's phases. It was thus supposed that the primary characteristic of all tides was a marked alteration in height from springs to neaps in the period of the synodic month.

In the Bay of Fundy, however, it is a noteworthy feature of the tide, that the variation in range from Perigee to Apogee is greater than the difference in range at mean springs and mean neaps. This shows the dominating influence of the moon's distance in that region, in the period of the anomalistic month. The strong tidal currents in the Bay of Fundy have also a variation in strength which follows the same law.

The data given below for St. John are from the record of the registering tide gauge there. The springs and neaps selected are in the months of August and September, when the Perigee and Apogee coincided with the full and change of the moon; and the intermediate neaps were closely at the moon's mean distance. The range given is the mean for two consecutive high waters and low waters, as the diurnal inequality is thus eliminated. The data for Cumberland Basin are from the day tides only, taken in 1870 by the Engineers of the proposed Baie Verte canal. The

months selected are October and November, when the above coincidence occurred in that year.

Description of Tide		St. John, N.B.		Cumberland Basin	
		Range in feet	Diff- erence	Range in feet	Diff- erence
At Perigee.	Range at Spring Tides	27.10)		48.20)	
At Apogee.	Range at Spring Tides	20.35)	6.75	35.55)	12.65
Spring range.	Mean of the above	23.72)		41.87)	
Neap range, at Moon's mean distance		17.43)	6.29	20.75)	12.12

The tidal differences throughout the Bay of Fundy are remarkably constant, however. The time-difference between Yarmouth and St. John, as obtained from two years of simultaneous observation, shows no appreciable variation in the course of the month or during the year. This indicates that the anomalistic feature is already fully developed at Yarmouth, where the range is only 16 feet; although the range itself continues to increase to 50 feet at the head of the bay.

Synodic month.—In dealing with the St. Lawrence tides, the endeavour was made to extend the region referred to Quebec to include the whole of the estuary of the St. Lawrence, and thus to save a reduction by harmonic analysis at an additional tidal station. It has proved better, however, to raise Father Point, in the middle of the estuary, to the rank of a principal station; but the earlier endeavour resulted in an elaborate comparison of the tides at Father Point and Quebec, which are 190 miles apart in one of the largest tidal estuaries in the world.

The crest of high water takes some $4\frac{1}{2}$ hours to run up the estuary from Father Point to Quebec; and low water still longer. Simultaneous observations at the two places during two complete years, summer and winter, afforded 1250 time-differences for high water, and 1280 for low water.

On making a close examination of the whole series of differences, it appeared that in the case of low water a double variation in the difference of time occurred; firstly in the period of the synodic month with the moon's phases, and secondly, in the period of the anomalistic month with the moon's

declination is high, an alternation in the time-interval and the height at successive tides, which accords with the moon's upper and lower transits, is almost everywhere appreciable; and in some regions this becomes so pronounced as to become the leading feature in the tide.

Northumberland strait affords a noteworthy example of a region dominated by the declination of the moon. At Charlottetown, where the range amounts to ten feet, the difference between the two tides of the 24 hours is greater than the difference between springs and neaps. A similar inequality affects the current in the strait. The difference of time between the turn of the current and the local tide is large; as the turn may take place within limiting periods of two hours relatively to the time of high water or low water; and the variation follows the change in the moon's declination. This is very confusing to the mariner, as the turn of the current is out of accord with the moon's phases, and has thus no fixed relation to the spring and neap tides. The greatest apparent irregularity is when the moon's declination is at its maximum; and this occurs sometimes at the spring tides and sometimes at the neaps. The ordinary navigator takes refuge in the conclusion that the currents are chiefly influenced by the wind. But the observations show that the apparent irregularities can be reduced to definite laws, which, although complex, are strictly astronomical in character.

It is evident that this feature in the tide develops during its progress. In the open Atlantic, as represented by the tide at Halifax, the diurnal inequality although appreciable is by no means noteworthy. But after entering the Gulf, as the tide proceeds westward towards Northumberland strait, the diurnal inequality becomes more pronounced; and it is further accentuated as the tide passes along the strait. It has therefore been found advisable to take Pictou as a secondary port of reference in the middle of the strait itself; as the inequality in the tide in the two directions can thus be better distributed.

Tide tables for St. Paul island in Cabot strait, the main entrance to the Gulf from the Atlantic, are calculated from the tidal constants resulting from harmonic analysis ; and to obtain data for Pictou, simultaneous observations were taken at the two places during four summer seasons, in 1896, 1897, 1901 and 1903. From these it was found that two series of variable differences in the time of the tide were required ; one for high water and the other for low water. The differences vary in accordance with the declination of the moon, and alternate with its upper and lower transits. It is also found necessary to allow for the variation in the range of the moon's declination in the 19-year cycle, by a modification in the series of differences used, according to the position of the year in the cycle. The table of variable differences which is used for a year when the range in declination is greatest, is appended as Table B.

It is very evident from the result of this investigation, that the declination element increases in the tide as it progresses from St. Paul island to Pictou. It becomes so dominant a feature that all variation in other lunar periods is relatively negligible in calculating the tides. It also continues to increase as the tide proceeds from Pictou to Charlottetown ; as the difference in time between the two places shows a further variation when the moon is near its maximum declination, north or south.

Why it should be that this declinational feature in the tide develops in Northumberland strait, while the lunar distance element or the anomalistic feature develops on the other side of Nova Scotia in the Bay of Fundy, we do not undertake to explain. A similar development of the declinational feature is also found in Belle Isle strait in both tide and current.

One curious result of a difference in the type of the tide in two neighboring regions, is the effect upon the current in a strait which connects two such regions. The Gut of Canso affords an example of this ; as the irregularity of its currents can only be attributed to the difference in the character of the tide itself, at the two ends of the Gut. The tide in the region of Northumberland strait at its northern end, shows a marked diurnal

inequality, which accords with the declination of the moon ; and as the declination-month is overrun by the period of the moon's phases, this diurnal inequality characterises sometimes the spring tides and sometimes the neaps. While these changes recur periodically at the northern end of the Gut, at the southern end the Atlantic tide maintains the usual variation in height from springs to neaps with great regularity. As the current through the Gut depends on tides which are so different in character at its two ends, it necessarily shows great complexity. Before this explanation was found by the investigation of the tides themselves, it was supposed that the currents were chiefly governed by the wind.

Reversal of the diurnal inequality.—Another interesting feature in the tides of the Gulf of St. Lawrence, is the reversal of the diurnal inequality between the entrance at Cabot strait and the opposite shore to the westward. We may designate as the open gulf coast, the northern part of the coast of New Brunswick and the north coast of Prince Edward island. The tides on these coasts can be referred to St. Paul island, provided that the difference in time is taken as earlier, or for the preceeding tide. Otherwise, the difference in the time of the tide varies so widely as to be practically valueless for purposes of computation. But with the above proviso, a constant difference in time is applicable.

These various examples which illustrate the dominant influence of some one element in the moon's motion in different regions, are also interesting in showing how unwise it is to theorize in carrying out tidal investigations, as the characteristics of each region must be ascertained by direct observation.

Character of the Pacific tide.—The tide of the Pacific can best be described as a declination-tide. Its leading feature is a pronounced diurnal inequality in time and height, which accords with the declination of the moon ; and it is also subject to an annual variation with the change in the declination of the sun. As the solar influence is unusually large in the Pacific the annual variation is the more accentuated.

When the moon is farthest north or south of the equator, the inequality between the two tides of the day is greatest ; and the long and short runs of the current occur. The extreme tides of the year necessarily occur at the nearest point to the solstices at which the moon reaches its maximum declination. On the other hand, the tides become equal when both the sun and moon are on the equator ; or when they are on opposite sides of the equator at distances north and south which are proportional to their respective effects.

The spring and neap tides are thus reduced to a secondary feature which is usually obscured by the stronger characteristics of the tide. The Establishment, which is so well marked in the Atlantic, is here almost illusory ; unless it is strictly reduced to equinoctial and equatorial conditions, in accordance with the definition used in France. It may still be convenient to speak of spring and neap tides, if they are understood to mean the two maxima and the two minima in range or in level which always occur in the period of the lunar month ; although as regards time of occurrence, they may be several days before or after the full or new moon, as these extremes are so largely due to the diurnal inequality.

A tide of this character is apt to be termed irregular by the mariner ; as the tropical or declination-month which is its governing period is less familiar and less noticeable than the synodic month of the moon's phases. It is evident, however, that this tide is perfectly astronomical ; and its analysis and prediction are just as definite as for any other type of tide.

We need not enter upon a description of the various types of tide met with on the Pacific coast, as it will suffice to say that for tidal purposes it is necessary to divide it into three or four regions, with a port of reference in each. For if the attempt is made to compute our British Columbia tides from tidal stations in the United States or Alaska, to the north or south, the resulting errors may exceed one or two hours in time.

Even in the extent of each region, there is still a variation in the tidal difference to allow for. The diurnal inequality can

best be dealt with by classifying the tides, and computing tidal differences separately for the higher high waters, the lower low waters, and the half tides, as those of intermediate range are termed. The differences are then found to be constant throughout a fairly extended region. But when comparisons between two places are obtained for a complete year, a marked annual variation comes to light.

It is thus best, when possible, to base the tidal difference upon a comparison for four months at the four quarters of the year. If these values are not sufficiently constant to give a satisfactory average, then a series of differences must be obtained for each month of the year.

On the open coast of the Pacific, a distinct annual variation has been found in the tidal difference between Port Simpson and Sitka in Alaska. Their latitudes are $54\frac{1}{2}^{\circ}$ and 57° N. The variation in the time-difference for high water, is from $1^h 10^m$ at the summer solstice to 51^m at the winter solstice.

In the Strait of Georgia, an annual variation is also distinct, although the time of the tide is nearly simultaneous throughout the strait. The principal station for that region is at Sand Heads, off the mouth of the Fraser River in the middle of the open strait.

Time of Tide in Baynes Sound, compared with Sand Heads. (Monthly Averages.)

	Dec. mins.	Jan. mins.	Feb. mins.	Mar. mins.	Apr. mins.	May mins.
Difference for H. W.	+ 3	- 1	0	+ 6	+ 8	+ 13
Difference for L. W.	- 2	- 2	- 1	0	0	+ 6

The following table shows the variation in the time of slack water in Active Pass when compared with the tide at Sand Heads. The amount of this variation is remarkable, as the two places are almost opposite on the two sides of the strait, and only 14 miles distant from each other.

Active Pass	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
H. W. Slack	h.m. 0:50	h.m. 0:50	h.m. 0:56	h.m. 1:06	h.m. 1:18	h.m. 1:28	h.m. 1:34	h.m. 1:34	h.m. 1:28	h.m. 1:18	h.m. 1:06	h.m. 0:56
Variation from mean	- 22	- 22	- 16	- 06	+ 06	+ 16	+ 22	+ 22	+ 16	+ 06	- 06	- 16

This variation in the course of the year is presumably in accord with the change in the declination of the sun, as this would correspond with the strongly marked effect which declination seems always to have upon the Pacific tides. It is not at all comparable to the variation in a river tide with the season of the year, due to change in the stage of the water, such as occurs in the St. Lawrence above Quebec, where allowance for the month of the year is required in the calculations.

It is quite possible that the annual variation is accentuated by the high latitude, where the direction of the attractive forces in summer is so different from their winter direction, when they become nearly horizontal at the upper or lower transit of the sun and moon. The study of the tides near the Arctic circle might therefore throw an important light on the subject of annual variation ; but as yet the data in those regions is very meagre.

Concluding remarks.—It may not be possible to explain satisfactorily the reason for the development of the different types of tide referred to, until our general tidal data are more complete. But the discussion of the subject may prove of value in its bearing on the study of the tides of the world in general. For it may never be practicable to obtain adequate tidal data for each harbour of the world independently, for which tide tables are required.

The most practical method will no doubt prove to be the classification of the tides into types ; and anything that tends to explain or distinguish these types is of value. A large saving in the work can also be effected by extending the region which can be referred to each principal tidal station, as a port of reference. This can be done by the use of variable tidal differences, as employed by this Survey, which the examples given may serve to illustrate. It will thus be possible to reduce to their least amount the labour and difficulty of securing sufficient tidal record for harmonic analysis ; as the record to serve this purpose, must be continuous day and night throughout the year, and entirely free from interruption.

This variation in the leading features of the tide also shows the importance of placing the principal stations at strategic points where the true type for the region can be secured. From this point of view, many of the important harbours of the world are very unsuitably situated to serve as ports of reference ; as they are at the mouths of rivers or in estuaries where disturbing local conditions are most felt. Some small island or isolated lighthouse, well situated as a strategic point, may serve better as a principal tidal station from which to calculate the tides for several important harbours in its region.

The change in the features of the tide, with its progress, points also to the advantage of a tidal station situated as centrally as possible in each region to minimize any variation in the two directions. The extent of the region that can be referred to the station must then be ascertained by observation and investigation, in accordance with the astronomical features which may there happen to be predominant ; and this will indicate the type of variable difference best adapted to extend the region as far as practicable. This will also enable the undue multiplication of principal tidal stations to be avoided.

Ottawa,

April 4, 1907.

TABLE A.—VARIATION IN THE SYNODIC AND ANOMALISTIC MONTHS.
 VARIABLE TIDAL DIFFERENCES FOR LOW WATER IN THE ST. LAWRENCE ESTUARY.
 (For Calculation of Low Water at Father Point, from Low Water at Quebec.)

Moon's Phases	Synodic Month			Anomalistic Month		
	No. of Tide	Difference for Low Water	Moon's Phases	No. of Tide	Correction in Minutes	Moon's Distance
		H. M.				
Full Moon	28	5 41	New Moon	26	-08	Perigee
	27	5 42		25	-08	
	26	5 43		24	-08	
	25	5 43		23	-08	
	24	5 42		22	-08	
	23	5 41		21	-08	
	22	5 40		20	-07	
	21	5 38		19	-07	
	20	5 35		18	-06	
	19	5 32		17	-06	
	18	5 29		16	-05	
	17	5 26		15	-05	
	16	5 22		14	-04	
	15	5 18		13	-03	
	14	5 15		12	-02	
	13	5 12		11	-01	
	12	5 11		10	0	
	11	5 11		9	+01	
	10	5 12		8	+02	
	9	5 15		7	+03	
	8	5 18		6	+04	
	7	5 22		5	+06	
	6	5 26		4	+07	
	5	5 29		3	+09	
	4	5 32		2	+10	
	3	5 35		1	+12	
	2	5 38		0	+14	Apogee
	1	5 40				
New Moon	0	5 41	Full Moon			
Total length of Synodic month = 57.06 tide-intervals.			Total length of Anomalistic month = 53.24 tide-intervals.			

TABLE B.—VARIATION IN THE DECLINATION-MONTH.

VARIABLE TIDAL DIFFERENCES FOR THE TIDE IN NORTHUMBERLAND STRAIT.

(For Calculation of Pictou tides from St. Paul Island.)

Differences to be added to the time of the tide at St. Paul Island, for result in Standard time.

In the numbering, the lower transit tides are enclosed in brackets. The term node is here used to indicate the point at which the moon crosses the equator, in passing from N. to S. declination, and S. to N.

For High Water			For Low Water		
Moon North	Difference	Moon South	Moon North	Difference	Moon South
Number of Tide after Zero declination		Number of Tide after Zero declination	Number of Tide after Zero declination		Number of Tide after Zero declination
Ascending Node	H M	Descending Node	Ascending Node	H M	Descending Node
(0)	1:41	0	0	1:27	(0)
1	1:41	(1)	(1)	1:27	1
(2)	1:41	2	2	1:27	(2)
3	1:41	(3)	(3)	1:27	3
(4)	1:41	4	4	1:27	(4)
5	1:41	(5)	(5)	1:27	5
(6)	1:41	6	6	1:27	(6)
7	1:41	(7)	(7)	1:27	7
(8)	1:37	8	8	1:27	(8)
9	1:41	(9)	(9)	1:18	9
(10)	1:28	10	10	1:27	(10)
11	1:41	(11)	(11)	1:10	11
(12)	1:21	12	12	1:27	(12)
13	1:41	(13)	(13)	1:04	13
(14)	1:16	14	14	1:27	(14)
15	1:41	(15)	(15)	1:00	15
(16)	1:13	16	16	1:27	(16)
17	1:41	(17)	(17)	1:00	17
(18)	1:13	18	18	1:27	(18)
19	1:41	(19)	(19)	1:00	19
(20)	1:16	20	20	1:27	(20)
21	1:41	(21)	(21)	1:04	21
(22)	1:21	22	22	1:27	(22)
23	1:41	(23)	(23)	1:10	23
(24)	1:28	24	24	1:27	(24)
25	1:41	(25)	(25)	1:18	25
(26)	1:37	26	26	1:27	(26)
Descending Node		Ascending Node	Descending Node		Ascending Node

It will be noticed in the table that the difference for high water is constant for all similar tides; that is, for upper transit tides when the moon is in north declination, and for lower transit tides when the moon is in south declination. Also, the least differences or minimum values for both high water and low water, occur at the third tide after the moon's maximum declination; which is the same as the interval at which spring tides occur after full and change of the moon.

This table is for a year of greatest range in the declination of the moon. Similar tables are used for years of least range, and for the mean between the two, in the course of the 19-year cycle.